ECML

2013

Advanced Math

DIRECTIONS: You **may** write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect or blank answers will **NOT** lower your score. You **MAY** use only a SAT I approved calculator on this test. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored, only in the case of a tie between the top scorers, and will not count as part of the team score.

The answer to part e) will always be **NG** for "Not Given". This is a viable answer and means that the correct answer is **not** one of the first four listed.

1) Evaluate: $\sqrt[3]{13\sqrt{2}\cos(\frac{3\pi}{4})-14}$

A) -1   B)   1   C)   -3   D)   3   E)   NG

2) Find the sum of the infinite series whose nth term is : $\dfrac{6^{n-1}}{11^{n}}$

A) $\dfrac{6}{5}$ B) $\dfrac{5}{6}$ C) $\dfrac{1}{4}$ D) $\dfrac{1}{5}$ E)   NG

3) If $\sin(A) = \dfrac{1}{4}$ and $\tan(A) < 0$, find $\sin(2A)$

A) $\dfrac{-\sqrt{15}}{8}$ B) $\dfrac{-\sqrt{17}}{8}$ C) $\dfrac{-\sqrt{15}}{4}$ D) $\dfrac{-\sqrt{17}}{4}$ E)   NG

4) Find the distance between the two points of intersection of the parabolas:

$y = x^2 - 2x + 4$ and $y = 2x^2 - x - 2$.

A) $10\sqrt{2}$ B) $2\sqrt{10}$ C) $10\sqrt{5}$ D) $5\sqrt{10}$ E)   NG

5) The polar equation $r = \dfrac{6}{2 - 3\cos(\theta)}$ is which of the following conic sections?

A) Circle B)   parabola   C)   ellipse   D)   hyperbola   E)   NG

6) What is the range of $(f \circ g)(x)$ if $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 3$?

A) $y \geq 0$ B) $y \geq 1$ C) $y \geq 2$ D) $y \geq 3$ E)   NG

7) Find the term containing no "x" in the expansion of: $(x^2 - \dfrac{a}{x^3})^{10}$

A) $-210a^4$ B) $210a^4$ C) $-252a^5$ D) $252a^5$ E)   NG

8) Solve for x: $2 + \log_3(x+2) = \log_3(4 - 5x)$

A) 1   B)   0   C)   -1   D)   -2   E)   NG

9) A parallelogram is drawn so that its vertices are the endpoints of the major and minor axis of the ellipse $9x^2 + 4y^2 - 18x + 24y + 9 = 0$. Find the area of the parallelogram.

A) 3   B)   6   C)   12   D)   24   E)   NG

10)   Effective January 27, 2013 the United States Postal Service increased its first class mail rates to: 46 cents for the first ounce or part thereof and 20 cents for each additional ounce or part thereof. Write the greatest integer function, INT[], which describes the cost (C), in cents, of mailing a first class letter whose weight is x ounces where x>0.

A) $C = 46 + 20INT|x+1|$ B) $C = 46 + 20INT|x-1|$ C) $C = 20 + 46INT|x+1|$
d) $C = 20 + 46INT|x-1|$ E)   NG

11)   If $f(x) = 3^x$, then f(x+2)-f(x+1)=

A) f(1)   B)   f(x)   C)   3f(x)   D)   6f(x)   E)   NG

12)   The product of two of the roots of $3x^3 - 9x^2 + 5x - 18 = 0$ is -2.   What is the third root?

A) 9   B)   -9   C)   3   D)   -3   E)   NG

13) Which of the following is NOT a sixth root of -1?

A) $\dfrac{-\sqrt{3}}{2}+\dfrac{1}{2}i$ B) $\dfrac{-1}{2}+\dfrac{\sqrt{3}}{2}i$ C) -i   D) i   E) NG

14) Which of the following describe(s) the roots of $x^4+7x^3+2x+1=0$

    I   Sum of the roots = -7

    II   There are no rational roots.

    III   There are no positive roots.

A) I only   B)   I and II only   C)   II and III only

d)   I and III only   E)   NG

15) In $\triangle ABC$, $\tan(A)=\dfrac{3}{4}$, $\cos(B)=\dfrac{5}{13}$, and AB=14. Find AC=

A) $\dfrac{40}{3}$ B)   11   C) $\dfrac{26}{3}$ D) cannot be uniquely determined E) NG

Tie Breaker. This question will only be scored to break a tie amongst the high scorers on this contest. It will not count as part of the team score.

Find the exact value of the $\sin(105°)$.